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Model Predictive Control of a Nonlinear System with Known Scheduling Variable

Mahmood Mirzaei* Niels Kjølstad Poulsen*
 Hans Henrik Niemann**

* *Department of Informatics and Mathematical Modeling, Technical University of Denmark, Denmark, (e-mail: mmir@imm.dtu.dk, nkp@imm.dtu.dk).*

** *Department of Electrical Engineering, Technical University of Denmark, Denmark, (e-mail: hhn@elektro.dtu.dk)*

Abstract: Model predictive control (MPC) of a class of nonlinear systems is considered in this paper. We will use Linear Parameter Varying (LPV) model of the nonlinear system. By taking the advantage of having future values of the scheduling variable, we will simplify state prediction. Consequently the control problem of the nonlinear system is simplified into a quadratic programming. Wind turbine is chosen as the case study and we choose wind speed as the scheduling variable. Wind speed is measurable ahead of the turbine, therefore the scheduling variable is known for the entire prediction horizon.

Keywords: Model predictive control, linear parameter varying, nonlinear systems, wind turbines, LIDAR measurements.

1. INTRODUCTION

Model predictive control (MPC) has been an active area of research and has been successfully applied on different applications in the last decades (Qin and Badgwell (1996)). The reason for its success is its straightforward ability to handle constraints. Moreover it can employ feedforward measurements in its formulation and can easily be extended to MIMO systems. However the main drawback of MPC was its on-line computational complexity which kept its application to systems with relatively slow dynamics for a while. Fortunately with the rapid progress of fast computations, better optimization algorithms, off-line computations using multi-parametric programming (Baotic (2005)) and dedicated algorithms and hardware, its applications have been extended to even very fast dynamical systems such as DC-DC converters (Geyer (2005)). Basically MPC uses a *model* of the plant to *predict* its future behavior in order to compute appropriate control signals to *control* outputs/states of the plant. To do so, at each sample time MPC uses the current measurement of outputs/states and solves an optimization problem. The result of the optimization problem is a sequence of control inputs of which only the first element is applied to the plant and the procedure is repeated at the next sample time with new measurements (Maciejowski (2002)). This approach is called receding horizon control. Therefore basic elements of MPC are: a model of the plant to predict its future, a cost function which reflects control objectives, constraints on inputs and states/outputs, an optimization algorithm and the receding horizon principle. Depending on the type of the model, the control problem is called linear MPC, hybrid MPC, nonlinear MPC etc. Nonlinear MPC

is normally computationally very expensive and generally there is no guarantee that the solution of the optimization problem is a global optimum. In this work we extend the idea of linear MPC using linear parameter varying (LPV) systems to formulate a tractable predictive control of nonlinear systems. To do so, we use future values of a disturbance to the system that acts as a scheduling variable in the model. However there are some assumptions that restrict our solution to a specific class of problems. The scheduling variable is assumed to be known for the entire prediction horizon. And the operating point of the system mainly depends on the scheduling variable.

2. PROPOSED METHOD

Generally the nonlinear dynamics of a plant could be modeled as the following difference equation:

$$x_{k+1} = f(x_k, u_k, d_k) \quad (1)$$

With x_k, u_k and d_k as states, inputs and disturbances respectively. Using the nonlinear model, the nonlinear MPC problem could be formulated as:

$$\min_u \quad \ell(x_N) + \sum_{i=0}^{N-1} \ell(x_{k+i|k}, u_{k+i|k}) \quad (2)$$

$$\text{Subject to} \quad x_{k+1} = f(x_k, u_k, d_k) \quad (3)$$

$$u_{k+i|k} \in \mathbb{U} \quad (4)$$

$$\hat{x}_{k+i|k} \in \mathbb{X} \quad (5)$$

Where ℓ denotes some arbitrary norm and \mathbb{U} and \mathbb{X} show the set of acceptable inputs and states. As it was mentioned because of the nonlinear model, this problem is computationally too expensive. One way to avoid this problem is to linearize around an equilibrium point of the system and use linearized model instead of the nonlinear model. However for some plants assumption of linear

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model does not hold for long prediction horizons as the plant operating point changes, for example based on some disturbances that act as a scheduling variable. An example could be a wind turbine for which wind speed acts as a scheduling variable and changes the operating point of the system.

2.1 Linear MPC formulation

The problem of linear MPC could be formulated as:

$$\min_{u_0, u_1, \dots, u_{N-1}} \|x_N\|_{Q_f} + \sum_{i=0}^{N-1} \|x_{k+i|k}\|_Q + \|u_{k+i|k}\|_R \quad (6)$$

$$\text{Subject to } x_{k+1} = Ax_k + Bu_k \quad (7)$$

$$u_{k+i|k} \in \mathbb{U} \quad (8)$$

$$\hat{x}_{k+i|k} \in \mathbb{X} \quad (9)$$

Assuming that we use norms 1, 2 and ∞ the optimization problem becomes convex providing that the sets \mathbb{U} and \mathbb{X} are convex. Convexity of the optimization problem makes it tractable and guarantees that the solution is the global optimum. The problem above is based on a single linear model of the plant around one operating point. However below we formulate our problem using linear parameter varying systems (LPV) in which the scheduling variable is known for the entire prediction horizon.

2.2 Linear Parameter Varying systems

Linear Parameter Varying (LPV) systems are a class of linear systems whose parameters change based on a scheduling variable. Study of LPV systems was motivated by their use in gain-scheduling control of nonlinear systems (Apkarian et al. (1995)). LPV systems are able to handle changes in the dynamics of the system by parameter varying matrices.

Definition (LPV systems) let $k \in Z$ denote discrete time. We define the following LPV systems:

$$x_{k+1} = A(\gamma_k)x_k + B(\gamma_k)u_k \quad (10)$$

$$A(\gamma_k) = \sum_{j=1}^{n_\gamma} A_j \gamma_{k,j} \quad B(\gamma_k) = \sum_{j=1}^{n_\gamma} B_j \gamma_{k,j} \quad (11)$$

Which $A(\gamma_k)$ and $B(\gamma_k)$ are functions of the scheduling variable γ_k . The variables $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, and $\gamma_k \in \mathbb{R}^{n_\gamma}$ are the state, the control input and the scheduling variable respectively.

2.3 Problem formulation

Using the above definition, the linear parameter varying (LPV) model of the nonlinear system with disturbances is of the following form:

$$\tilde{x}_{k+1} = A(\gamma_k)\tilde{x}_k + B(\gamma_k)\tilde{u}_k + B_d(\gamma_k)\tilde{d}_k \quad (12)$$

This model is formulated based on deviations from the operating point. However we need the model to be formulated in absolute values of inputs, states and disturbances. Because in our problem the steady state point changes as a function of the scheduling variable and we need to introduce a variable to capture its behavior. In order to rewrite the state space model in the absolute form we use:

$$\tilde{x}_k = x_k - x_k^* \quad (13)$$

$$\tilde{u}_k = u_k - u_k^* \quad (14)$$

$$\tilde{d}_k = d_k - d_k^* \quad (15)$$

x_k^* , u_k^* and d_k^* are values of states, inputs and disturbances at the operating point. Therefore the LPV model becomes:

$$x_{k+1} = A(\gamma_k)(x_k - x_k^*) + B(\gamma_k)(u_k - u_k^*) + B_d(\gamma_k)(d_k - d_k^*) + x_{k+1}^* \quad (16)$$

Which could be written as:

$$x_{k+1} = A(\gamma_k)x_k + B(\gamma_k)u_k + B_d(\gamma_k)d_k + \lambda_k \quad (17)$$

with

$$\lambda_k = x_{k+1}^* - A(\gamma_k)x_k^* - B(\gamma_k)u_k^* - B_d(\gamma_k)d_k^* \quad (18)$$

Now having the LPV model of the system we proceed to compute state predictions. In linear MPC predicted states at step n is:

$$x_{k+n} = A^n x_k + \sum_{i=0}^{n-1} A^i B u_{k+(n-1)-i} \quad (19)$$

for $n = 1, 2, \dots, N$

However in our method the predicted state is also a function of scheduling variable $\Gamma_n = (\gamma_{k+1}, \gamma_{k+2}, \dots, \gamma_{k+n})^T$ for $n = 1, 2, \dots, N-1$ and we assume that the scheduling variable is known for the entire prediction. Therefore the predicted state could be written as:

$$x_{k+1}(\gamma_k) = A(\gamma_k)x_k + B(\gamma_k)u_k + B_d(\gamma_k)d_k + \lambda_k \quad (20)$$

And for $n \in \mathbb{Z}, n \geq 1$:

$$\begin{aligned} x_{k+n+1}(\Gamma_n) &= \prod_{i=n}^0 A(\gamma_{k+i})x_k \\ &+ \sum_{j=0}^{n-1} \left(\prod_{i=n-j}^1 A(\gamma_{k+i}) \right) B(\gamma_{k+j})u_{k+j} \\ &+ \sum_{j=0}^{n-1} \left(\prod_{i=n-j}^1 A(\gamma_{k+i}) \right) B_d(\gamma_{k+j})d_{k+j} \\ &+ \sum_{j=0}^{n-1} \left(\prod_{i=n-j}^0 A(\gamma_{k+i}) \right) \lambda_{k+(n-1)-j} \\ &+ B(\gamma_{k+n})u_{k+n} + B_d(\gamma_{k+n})d_{k+n} + \lambda_{k+n} \end{aligned} \quad (21)$$

Using the above formulas we write down the stacked predicted states which becomes:

$$X = \Phi(\Gamma)x_k + \mathcal{H}_u(\Gamma)U + \mathcal{H}_d(\Gamma)D + \Phi_\lambda(\Gamma)\Lambda \quad (22)$$

with

$$X = (x_{k+1} \ x_{k+2} \ \dots \ x_{k+N})^T \quad (23)$$

$$U = (u_k \ u_{k+1} \ \dots \ u_{k+N-1})^T \quad (24)$$

$$D = (d_k \ d_{k+1} \ \dots \ d_{k+N-1})^T \quad (25)$$

$$\Gamma = (\gamma_k \ \gamma_{k+1} \ \dots \ \gamma_{k+N-1})^T \quad (26)$$

$$\Lambda = (\lambda_k \ \lambda_{k+1} \ \dots \ \lambda_{k+N-1})^T \quad (27)$$

In order to summarize formulas for matrices Φ , Φ_λ , \mathcal{H}_u and \mathcal{H}_d , we define a new function as:

$$\psi(m, n) = \prod_{i=m}^n A(\gamma_{k+i}) \quad (28)$$

Therefore the matrices become:

$$\Phi(\Gamma) = \begin{pmatrix} \psi(1,1) \\ \psi(2,1) \\ \psi(3,1) \\ \vdots \\ \psi(N,1) \end{pmatrix}$$

$$\Phi_\lambda(\Gamma) = \begin{pmatrix} I & 0 & 0 & \dots & 0 \\ \psi(1,1) & I & 0 & \dots & 0 \\ \psi(2,1) & \psi(2,2) & I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \psi(N-1,1) & \psi(N-1,2) & \psi(N-1,3) & \dots & I \end{pmatrix}$$

$$\mathcal{H}_u(\Gamma) = \begin{pmatrix} B(\gamma_k) & 0 & \dots & 0 \\ \psi(1,1)B(\gamma_k) & B(\gamma_{k+1}) & \dots & 0 \\ \psi(2,1)B(\gamma_k) & \psi(2,2)B(\gamma_{k+1}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \psi(N-1,1)B(\gamma_k) & \psi(N-1,2)B(\gamma_{k+1}) & \dots & B(\gamma_{N-1}) \end{pmatrix}$$

$$\mathcal{H}_d(\Gamma) = \begin{pmatrix} B_d(\gamma_k) & 0 & \dots & 0 \\ \psi(1,1)B_d(\gamma_k) & B_d(\gamma_{k+1}) & \dots & 0 \\ \psi(2,1)B_d(\gamma_k) & \psi(2,2)B_d(\gamma_{k+1}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \psi(N-1,1)B_d(\gamma_k) & \psi(N-1,2)B_d(\gamma_{k+1}) & \dots & B_d(\gamma_{N-1}) \end{pmatrix}$$

After computing the state predictions as functions of control inputs (22), we can write down the optimization problem similar to a linear MPC problem as a quadratic program:

$$\begin{aligned} \min_U \quad & X^T Q X + U^T R U \\ \text{Subject to:} \quad & U \in \mathbb{U} \\ & X \in \mathbb{X} \end{aligned} \quad (29)$$

3. CASE STUDY

The case study here is a wind turbine. Wind turbine control is a challenging problem as the dynamics of the system changes based on wind speed which has a stochastic nature. The method that we propose here is to use wind speed as a scheduling variable. With the advances in LIDAR technology (Harris et al. (2006)) it is possible to measure wind speed ahead of the turbine and this enables us to have the scheduling variable of the plant for the entire prediction horizon.

3.1 Modeling

Nonlinear model For modeling purposes, the whole wind turbine can be divided into 4 subsystems: Aerodynamics subsystem, mechanical subsystem, electrical subsystem and actuator subsystem. The aerodynamic subsystem converts wind forces into mechanical torque and thrust on the rotor. The mechanical subsystem consists of drivetrain, tower and blades. Drivetrain transfers rotor torque to electrical generator. Tower holds the nacelle and withstands the thrust force. And blades transform wind forces into torque and thrust. The generator subsystem converts mechanical energy to electrical energy and finally the blade-pitch and generator-torque actuator subsystems are part of the control system. To model the whole wind turbine, models of these subsystems are obtained and at the end they are connected together. A wind model is obtained and augmented with the wind turbine model to be used for wind speed estimation. Figure 1 shows the

basic subsystems and their interactions. The dominant dynamics of the wind turbine come from its flexible structure. Several degrees of freedom could be considered to model the flexible structure, but for control design mostly just a few important degrees of freedom are considered. In figure 2 basic degrees of freedom which are normally being considered in the design model are shown. However in this work we only consider two degrees of freedom, namely the rotational degree of freedom (DOF) and drivetrain torsion. Nonlinearity of the wind turbines mostly comes from its aerodynamics. Blade element momentum (BEM) theory (Hansen (2008)) is used to numerically calculate aerodynamic torque and thrust on the wind turbine. This theory explains how torque and thrust are related to wind speed, blade pitch angle and rotational speed of the rotor. In steady state, i.e. disregarding dynamic inflow, the following formulas can be used to calculate aerodynamic torque and thrust.

$$Q_r = \frac{1}{2} \frac{1}{\omega_r} \rho \pi R^2 v_e^3 C_p(\theta, \omega, v_e) \quad (30)$$

$$Q_t = \frac{1}{2} \rho \pi R^2 v_e^2 C_t(\theta, \omega, v_e) \quad (31)$$

In which Q_r and Q_t are aerodynamic torque and thrust, ρ is the air density, ω_r is the rotor rotational speed, v_e is the effective wind speed, C_p is the power coefficient and C_t is the thrust force coefficient. The absolute angular position of the rotor and generator are of no interest to us, therefore we use $\psi = \theta_r - \theta_g$ instead which is the drivetrain torsion. Having aerodynamic torque and modeling drivetrain with a simple mass-spring-damper, the whole system equation with 2 degrees of freedom becomes:

$$J_r \dot{\omega}_r = Q_r - c(\omega_r - \frac{\omega_g}{N_g}) - k\psi \quad (32)$$

$$(N_g J_g) \dot{\omega}_g = c(\omega_r - \frac{\omega_g}{N_g}) + k\psi - N_g Q_g \quad (33)$$

$$\dot{\psi} = \omega_r - \frac{\omega_g}{N_g} \quad (34)$$

$$P_e = Q_g \omega_g \quad (35)$$

In which J_r and J_g are rotor and generator moments of inertia, ψ is the drivetrain torsion, c and k are the drivetrain damping and stiffness factors respectively lumped in the low speed side of the shaft and P_e is the generated electrical power. For numerical values of these parameters and other parameters given in this paper, we refer to (Jonkman et al. (2009)).

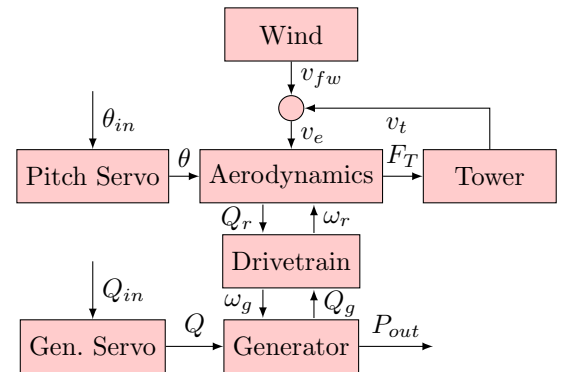


Fig. 1. Wind turbine subsystems

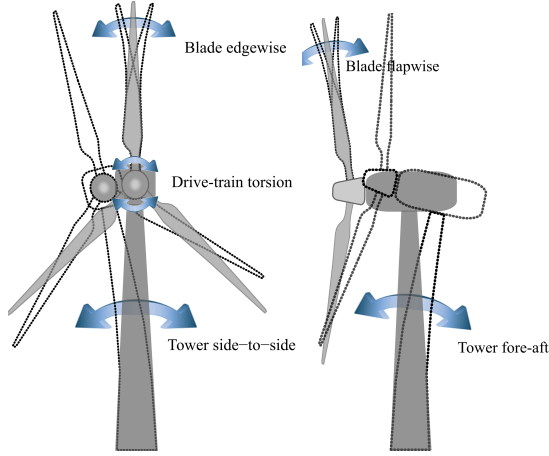


Fig. 2. Basic degrees of freedom

Linearized model As it was mentioned in the previous section, wind turbines are nonlinear systems. A basic approach to design controllers for nonlinear systems is to linearize them around some operating points. For a wind turbine, the operating points on the quasi-steady C_p and C_t curves are nonlinear functions of rotational speed ω_r , blade pitch θ and wind speed v . To get a linear model of the system we need to linearize around these operating points. Rotational speed and blade pitch are measurable with enough accuracy, however this is not the case for the effect of wind on the rotor. Wind speed changes along the blades and with azimuth angle (angular position) of the rotor. This is because of wind shear and tower shadow and stochastic spatial distribution of the wind field. Therefore a single wind speed does not exist to be used and measured for finding the operating point. We bypass this problem by defining a fictitious variable called effective wind speed (v_e) which shows the effect of wind in the rotor disc on the wind turbine. In our two DOFs model only the aerodynamic torque (Q_r) and electric power (P_e) are nonlinear. Taylor expansion is used to linearize them.

$$\Delta Q_r(\omega, \theta, v_e) = \underbrace{\frac{\partial Q_r}{\partial \omega} \Delta \omega}_a + \underbrace{\frac{\partial Q_r}{\partial \theta} \Delta \theta}_{b_1} + \underbrace{\frac{\partial Q_r}{\partial v_e} \Delta v_e}_{b_2} \quad (36)$$

$$\Delta P_e = \underbrace{\frac{\partial P_e}{\partial \omega_g} \Delta \omega_g}_{Q_{g0}} + \underbrace{\frac{\partial P_e}{\partial Q_g} \Delta Q_g}_{\omega_{g0}} \quad (37)$$

For the sake of simplicity in notations we use Q_r , P_e , θ , ω and v_e instead of ΔQ_r , ΔP_e , $\Delta \theta$, $\Delta \omega$ and Δv_e around the operating points from now on. Using the linearized aerodynamic torque, the 2 DOFs linearized model becomes:

$$\dot{\omega}_r = \frac{a-c}{J_r} \omega_r + \frac{c}{J_r} \omega_g - \frac{k}{J_r} \psi + b_1 \theta + b_2 v_e \quad (38)$$

$$\dot{\omega}_g = \frac{c}{N_g J_g} \omega_r - \frac{c}{N_g^2 J_g} \omega_g + \frac{k}{N_g J_g} \psi - \frac{Q_g}{J_g} \quad (39)$$

$$\dot{\psi} = \omega_r - \frac{\omega_g}{N_g} \quad (40)$$

$$P_e = Q_{g0} \omega_g + \omega_{g0} Q_g \quad (41)$$

A more detailed description of the model and linearization is given in (Mirzaei et al. (2011)).

LPV model Collecting all the discussed models, matrices of the state space model become:

$$A(\gamma) = \begin{pmatrix} \frac{a(\gamma)-c}{J_r} & \frac{c}{J_r} & -\frac{k}{J_r} \\ \frac{c}{N_g J_g} & -\frac{c}{N_g^2 J_g} & \frac{k}{N_g J_g} \\ 1 & -1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & Q_{g0} & 0 \end{pmatrix} \quad (42)$$

$$B(\gamma) = \begin{pmatrix} b_1(\gamma) & 0 \\ 0 & -\frac{1}{J_g} \\ 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \omega_{g0} \end{pmatrix} \quad (43)$$

In which $x = (\omega_r \ \omega_g \ \psi)^T$, $u = (\theta \ Q_g)^T$ and $y = (\omega_r \ \omega_g \ P_e)^T$ are states, inputs and outputs respectively. In the matrix B , parameter b_1 is uncertain. Therefore the uncertain linear state space model becomes:

$$\begin{aligned} \dot{x} &= A(\gamma)x + B(\gamma)u \\ y &= Cx + Du \end{aligned}$$

3.2 Control objectives

The most basic control objective of a wind turbine is to maximize captured power during the life time of the wind turbine. This means trying to maximize captured power when wind speed is below its rated value. This is also called maximum power point tracking (MPPT). However when wind speed is above rated, control objective becomes regulation of the outputs around their rated values while trying to minimize dynamic loads on the structure. These objectives should be achieved against fluctuations in wind speed which acts as a disturbance to the system. In this work we have considered operation of the wind turbine in above rated (full load region). Therefore we try to regulate rotational speed and generated power around their rated values and remove the effect of wind speed fluctuations.

3.3 Offset free control

Persistent disturbances and modeling error can cause an offset between measured outputs and desired outputs. To avoid this problem we have employed an offset free reference tracking approach (see Muske and Badgwell (2002) and Pannocchia and Rawlings (2003)). Our RMPC solves the regulation problem around the operating point. However we regulate around the operating point (x_k^* and u_k^*) which results in offset from desired outputs. To avoid this problem in our control algorithm we shift origin in our regulation problem to x_k^0 and u_k^0 instead. In order to find new origins, we have augmented linear model of the plant with a disturbance model that adds fictitious disturbances to the system. The fictitious disturbances compensate the difference between measured outputs and desired outputs. State space model of the augmented system is:

$$\tilde{x}_{k+1} = \tilde{A} \tilde{x}_k + \tilde{B} u_k \quad (44)$$

$$y_k = \tilde{C} \tilde{x}_k + D u_k \quad (45)$$

in which the augmented state and matrices are:

Table 1. Performance comparison between gain scheduling approach and linear MPC

Parameters	Proposed approach	Linear MPC
SD of ω_r (RPM)	0.111	0.212
SD of P_e (Watts)	4.686×10^4	8.048×10^4
Mean value of P_e (Watts)	4.998×10^6	4.998×10^6
SD of pitch (degrees)	2.67	2.95
SD of shaft moment (N.M.)	256	293

$$\tilde{x}_k = \begin{pmatrix} \hat{x}_{k+1} \\ \hat{d}_{k+1} \\ \hat{p}_{k+1} \end{pmatrix} \quad \tilde{A} = \begin{pmatrix} A & B_d & 0 \\ 0 & A_d & 0 \\ 0 & 0 & A_p \end{pmatrix} \quad (46)$$

$$\tilde{B} = (B \ 0 \ 0)^T \quad \tilde{C} = (C \ 0 \ C_p) \quad (47)$$

\hat{x}_k, \hat{d}_k and \hat{p}_k are system states, input/state and output disturbances respectively. (A, B, C, D) are matrices of the linearized model, B_d and C_p show effect of disturbances on states and outputs respectively. A_d and A_p show dynamics of input/state and output disturbances. For more information and how to choose these matrices we refer to (Muske and Badgwell (2002)) and (Pannocchia and Rawlings (2003)). Since the disturbances are not measurable, an extended Kalman filter is designed to estimate them. The estimated disturbances are used to remove any offset between desired outputs and measured outputs. Based on this model and estimated disturbances, x_k^0 and u_k^0 which are offset free steady state input and states can be calculated:

$$\begin{pmatrix} A - I & B \\ C & D \end{pmatrix} \begin{pmatrix} x_k^0 \\ u_k^0 \end{pmatrix} = \begin{pmatrix} -B_d \hat{d}_k \\ -C_p \hat{p}_k \end{pmatrix} \quad (48)$$

After calculating these values, we simply replace x_k^* and u_k^* in (18) with x_k^0 and u_k^0 which results in:

$$\lambda_k = x_{k+1}^0 - A(\gamma_k)x_k^0 - B(\gamma_k)u_k^0 - B_d(\gamma_k)d_k^* \quad (49)$$

4. SIMULATIONS

In this section simulation results for the obtained controller are presented. The controller is implemented in MATLAB and is tested on a full complexity FAST (Jonkman and Jr. (2005)) model of the reference wind turbine (Jonkman et al. (2009)). Simulations are done with realistic turbulent wind speed, with Kaimal model (iec (2005)) as the turbulence model and TurbSim (Jonkman (2009)) is used to generate wind profile. In order to stay in the full load region, a realization of turbulent wind speed is used from category *C* of the turbulence categories of the IEC 61400-1 (iec (2005)) with 18m/s as the mean wind speed.

4.1 Stochastic simulations

In this section simulation results for a stochastic wind speed is presented. Control inputs which are pitch reference θ_{in} and generator reaction torque reference Q_{in} along with system outputs which are rotor rotational speed ω_r and electrical power P_e are plotted in figures 3-6 (red-dashed lines are results of linear MPC and solid blue lines show the results of the proposed approach.) Simulation results show good regulations of generated power and rotational speed. Table 1 shows a comparison of the results between the proposed approach and MPC approach based on linearization at each sample point (Henriksen (2007)).

As it could be seen from the table and figures, the proposed approach gives better regulation on rotational speed and generated power (smaller standard deviations) while maintaining a smaller shaft moment and pitch activity.

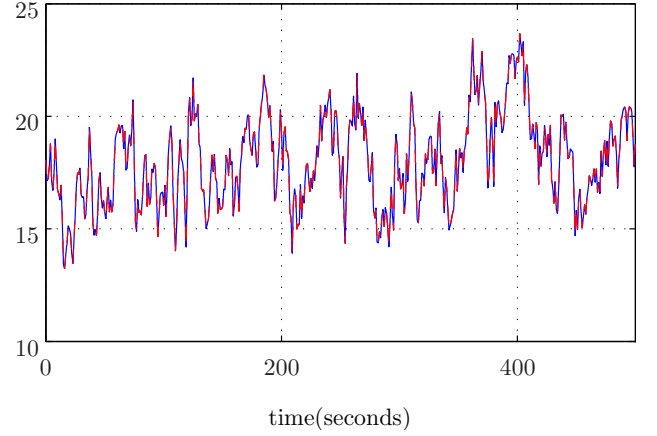


Fig. 3. Blade-pitch reference (degrees, red-dashed line is linear MPC and solid blue line is the proposed approach)

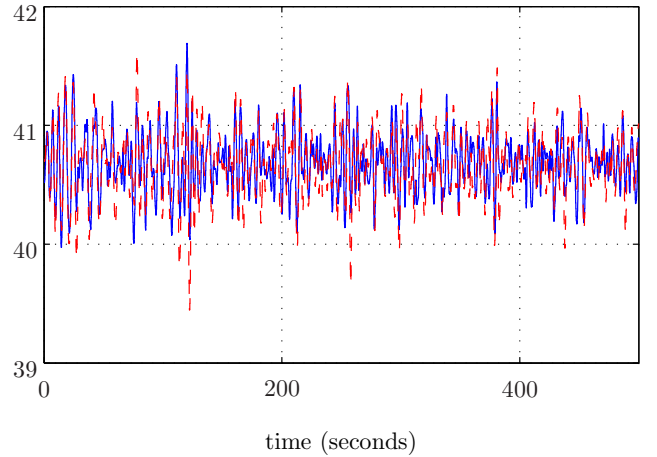


Fig. 4. Generator-torque reference (kNM, red-dashed line is linear MPC and solid blue line is the proposed approach)

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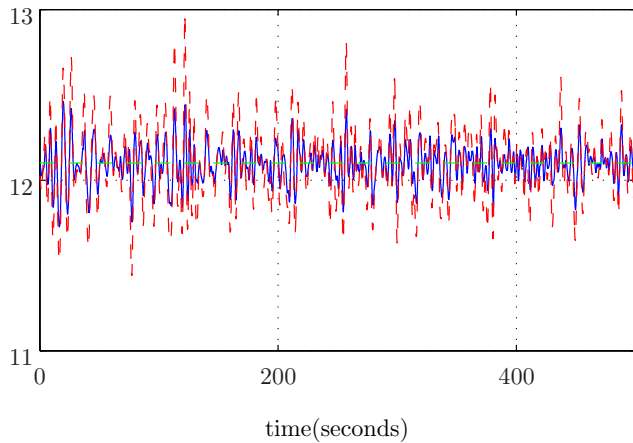


Fig. 5. Rotor rotational speed (ω_r , rpm, red-dashed line is linear MPC and solid blue line is the proposed approach)

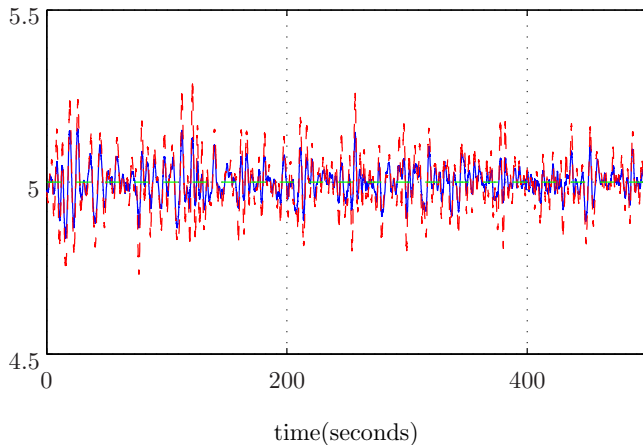


Fig. 6. Electrical power (mega watts, red-dashed line is linear MPC and solid blue line is the proposed approach)

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